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INVENTORY MODEL (nQ, R, T) RANDOM SUPPLY, CONSTANT LEAD TIMES AND EXPONENTIAL BACK ORDER COSTS

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Abstract

This paper derives the inventory costs for the model when the backorder costs are exponential, lead time is constant and supply is random.
It derives firstly the inventory cost for fixed constant lead times and exponential backorder costs. The random supply cost is obtained by averaging the result over the states of supply.
Demand during lead time is normal and supply is assumed to be a gamma variate
It derives the expected backorder costs, the expected number of backorders at any point in time and the probability of a stockout.

Introduction

We assume that demand during them lead time follows a normal distribution and supply is gamma variate. The exponential backorder $C_\beta(t)$ b_1 exp(b_2t) where t is the backorder length of time.
Firstly, we derive the inventory costs for constant lead time and exponential backorders. Then we then average the results over the states of supply by integral calculus.

Literature Review

Emal Arikan (2005) his thesis assumed three different supply process eg. Under the all-or-nothing type supply process and partially available supply process, the structure of optional policy is proved to be a base stock policy. He shows that a simple base stock policy is not optional using binomially distributed supply process. Muk Hopadhayay (2007) indicates that when order fluctuations and backorder fluctuations are observed other costs are incurred. If it is decided to reduce the aggregate inventory, the average production batch size should be decreased in order to maintain a balanced inventory. Over a range, the cost curve may be approximated by a quadratic.
Sicila (2012) derives a model where backorder demand rate is exponentially decreasing with the waiting time. Hardly and within discusses the model when lead time is constant.
C_β (t) = b_1 exp (b_2t) and demand follows a normal distribution and σ^2z in the variance of demand over a period z.
If the inventory position of the system immediately after review at time t, is R+Y, the expected backorder cost at time t is

$$= \frac{1}{Q} \int_0^Q D \int_0^L D \int_0^t C_\beta \; (t - z) \; \frac{1}{\sqrt{\sigma^2 L}} \; g\left(\frac{R+Y-DZ}{\sqrt{\sigma^2 L}}\right) dz \; dt \; di \; Y$$

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Where $\frac{1}{\sqrt{\sigma^2 L}}\ g\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right)$ is the normal distribution. (1)

Similarly the expected backorder costs at time t + L + T

$$= \frac{1}{Q}\int_0^Q D \int_0^{L+T} D \int_0^t C_\beta\ (t-z)\ \frac{1}{\sqrt{\sigma^2 t}}\ g\left(\frac{R+Y-DZ}{\sqrt{\sigma^2 t}}\right) dz\ dt\ d\ Y \quad (2)$$

Where C_β (L-z) = b_1 exp (b_2 (L-z))

Let G_1 (Q, R, L) = $\frac{1}{Q}\int_0^Q D \int_0^L D \int_0^t \frac{b_1 \exp b_2\ (L-z)}{\sqrt{\sigma^2 t}}\ g\left(\frac{R+Y-DZ}{\sqrt{\sigma^2 t}}\right) dz\ dt\ d\ Y$ (3)

Integrating G_1 (Q, R, L) with respect to Q and rearranging we have

$$G_2\ (R, L) = \int_0^L \frac{b_1}{b_2}\left(\exp - (\frac{b_2 R}{D} - b_2 t - \frac{\sigma^2 t\, b_2^2}{2\, D^2}\right) *$$

$$\left(F\left(\frac{R - \frac{\sigma^2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right) - F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) dt\right) \quad (4)$$

Then G_1 (R_1 L) = $\frac{1}{Q}\left(G_2\ (R, L) - G_2(R + Q_1 L)\right)$ (5)

Re-arranging the exponential terms of G_2 (R, L)

$$G_1\ (R_1\ L) = D\int_0^L \frac{b_1}{b_2} \exp\left(t\left(\frac{\sigma^2 b_2^2}{2\, D^2} + b_2\right) - \frac{b_2 R}{D}\right) F\left(\frac{R - \frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right)$$

Integrating by parts

$$\frac{1}{D}\ G_2\ (R, L) = \frac{b_1}{b_2}\left(\frac{2\, D^2}{\sigma^2\, b_2 + 2D^2\, b^2}\right)\left[\exp\left(t\, (\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b^2}{D}\ R\right)$$

$$F\left(R - \frac{\frac{\sigma^2}{D} t\, b_2 - Dt}{\sqrt{\sigma^2 \epsilon}}\right)\Bigg]\ \frac{b_1}{2\, b_2}\left(\frac{2\, D^2}{\sigma^2 + 2\, D^2\, b_2}\right)\int_0^L esp\left(t\left(\frac{\sigma^2 b_2^2}{2\, D_2} + b_2\right)\right.$$

$$\left. - \frac{b_2 R}{D}\right) \exp - \frac{1}{2}\left(R - \frac{\frac{\sigma^2 b_2 Z}{D}}{\sqrt{\sigma^2 t}} - Dz\right)\left(\frac{D^2 + \sigma^2 b_2}{D\sqrt{\sigma^2 t}} + \frac{R}{\sigma t^{3}/_{2}}\right)\ dz$$

$$-\frac{b_1}{b_2}\int_0^L F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\ dt$$

Let Z_n (x, T) = $\int_0^T t^n \exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\ dt$

and the fact that

$$Z_{n+1}\ (x_1\ T) = \frac{-\,2\,\sigma^2\, T^{n+1}}{D^2\sqrt{2\,\pi\sigma^2\, T}} \exp\left(\frac{x-DT}{\sqrt{\sigma^2\, T}}\right)^2 - \frac{\sigma^2\ (2n+1)}{2\, D^2} Z_n\ (x, T)$$

$$+ \frac{x^{\ 2}}{D^2}\ Z_{n-1}(x, T)$$

$$Z_{n+1}\ (x, T) = \frac{-\,2\,\sigma^2\, T}{D^2\sqrt{2\pi\,\sigma^2\, T}} \exp - ½\left(\frac{x-DT}{\sqrt{\sigma^2\, T}}\right) + \frac{\sigma^2}{D^2} Z_0\ (x^2, T)$$

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$+ \frac{x^2}{D^2} \, Z_{-1} (x, T)$

We obtain

$$G_2 (R, L) = \left(\frac{2 D^3 b_1}{\sigma^2 b_2^3 + 2D^2 b_2^2}\right) \exp \left[L \left(\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) \exp\left(-\frac{b^2}{D}\right)\right.\right.$$

$$F\left(\frac{R-DL\left(1+\frac{\sigma^1 b_2}{D^2}\right)}{\sqrt{\sigma^2 L}}\right) - \frac{b_1}{b_2} F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left[DL - R - \frac{\sigma^2}{2D} + \frac{D}{b_2}\right]$$

$$\frac{\sigma^4 b_1 b_2^2}{-2)(\sigma^2 b_2^2 + 2 D^2 b_2) b_2} \exp\left(\frac{2RD}{\sigma^2}\right) F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) - 2\frac{\sqrt{\sigma^2 L}}{b_2} \, b_1 g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

Hence

$$G_1 (Q, R, L) = ((G_2 (R_1 L) - G_2 (R + Q, L)) / Q$$

Expected backorder cost at time t + L averaging over the states of Y

$$= \frac{1}{Q} \left(G_2 (R, L) - G_2 (R + Q, L)\right) \quad (6)$$

and the expected backorder cost at t + L + T averaging over the states of Y

$$= \frac{1}{Q} \left(G_2 (R, L + T) - G_2 (R + Q, L + T)\right)$$

Hence the expected backorder cost per year $G_3 (Q, R, T)$

$$G_3(Q, R, T) = \frac{1}{Q} \left(G_2 (R_1 L + T) - G_2 (R_1 L) - G_2 (R + Q_1 L + T) + G_2 (R + Q, L)\right)$$

Let B(Q, R, T) be the expected number of backorder, at any point in time.

At anytime $t + L + \epsilon$ between t + L and t + L + T, the expected number of backorder on the books when the inventory position was R + Y immediately after review at time t

$$B(Q, R, T) = \frac{1}{QT} \int_Q^{R+Q} \int_L^{L+T} \int_w^{\infty} (x - w) g\left(\frac{x \, Dt}{\sqrt{\sigma^2 t}}\right) dx \, dt \, dw$$

Nothing that

$$\int_w^{\infty} (x - w) g (x \, Dt) dx = \sigma^2 t \, g (w, Dt) - (w - Dt) F\left(\frac{w - Dt}{\sqrt{\sigma^2 t}}\right)$$

Substituting in B(Q, R, T) and simplifying we have

$$B(Q, R, T) = \frac{1}{QT} \int_Q^{R+Q} \int_0^L (\sigma^2 t \, g (w, Dt) - (w - Dt) F (w, Dt)) \, dt \, dw$$

$$-\frac{1}{QT} \int_Q^{R+Q} \int_0^L (\sigma^2 t \, g (w, Dt) - (w - Dt) F (w, Dt)) \, dt \, dw \quad (8)$$

Note that

$$\int_0^T g (x, Dt) dt = \frac{1}{D} \left(F\left(\frac{x - Dt}{\sqrt{\sigma^2 t}}\right) - \exp\left(\frac{2Dx}{\sigma^2}\right) F\left(\frac{x + Dt}{\sqrt{\sigma^2 t}}\right)\right)$$

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$$\int_0^T tF\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)dt = \frac{1}{2}\left(T^2 - \frac{x^2}{D^2} - \frac{2\,\sigma^2 x}{D} - \frac{3\,\sigma^4}{2D^4}\right)F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)$$
$$+\frac{\sqrt{\sigma^2 T}}{2D^2}\left(DT + \frac{3\sigma^2}{D} - x\right)\frac{1}{\sqrt{2\pi\,\sigma^2 T}}\exp - \frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2$$
$$-\frac{\sigma^2}{2D^3}\left(x - \frac{3\sigma^2}{2D}\right)\exp\left(\frac{2Dx}{2}\right)F\left(\frac{x+DT}{\sqrt{\sigma^2 T}}\right)$$

Note that

$$\int_0^T F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)dt = \left(T - \frac{x}{D} - \frac{\sigma^2}{2D^2}\right)F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)$$

$$+\frac{\sqrt{\sigma^2 T}}{D\sqrt{2\pi}}\exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 T}}\right)^2 + \frac{\sigma^2}{2D^2}\exp\left(\frac{2Dx}{\sigma^2}\right)F\left(\frac{x+DT}{\sqrt{\sigma^2 T}}\right) \quad (9)$$

We obtain

$$B(Q,R,T) = \frac{1}{QT}\left(G_4\,(R+L,T) - G_4(R,L) - G_4(R+Q,L+T)\right) + G_4(R+Q,L)\Big)$$

Where $G_4\,(R_1T)$

$$G_4\,(R_1T) = \left(\frac{D^2T^3}{6} - \frac{\sigma^4 R}{4D^3} - \frac{DT^2R}{2} - \frac{\sigma^2R^2}{4D^2} + \frac{\sigma^2T^2}{4} + TR^2 - \frac{R^3}{6D} - \frac{\sigma^6}{8D^4}\right)F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)$$
$$+\left(\frac{DT^{5/2}\sigma}{6} - \frac{\sigma T^{3/2}R}{3} + \frac{\sigma T^{\frac{1}{2}}R^2}{6D} + \frac{\sigma^3T^{3/2}}{12D} + \frac{\sigma^3T^{\frac{1}{2}}R}{4D^2} + \frac{\sigma^5T^{1/2}}{4D^3}\right)g\left(\frac{R-DT}{\sigma^2 T}\right) + \frac{\sigma^6}{8D^4}\exp\left(\frac{2DR}{\sigma^2}\right)F\left(\frac{R+DT}{\sqrt{\sigma^2 T}}\right)$$

Let POR be the probability of a stockout, at any time between t+L and t+L+T that is the probability that demand exceeds R+Y at time t+L+∈.

$$= \int_{R+Y}^{\infty} g\,(x, D(L{+}\in)\;dx$$

Probability of a stockout

$$POR = \int_L^{L+T}\int_{R+Y}^{\infty} g\,(x, D\in)dx\;d\in$$

Averaging the states of Y and integrating with respect to π

$$POR = \frac{1}{Q}\int_0^Q\int_L^{L+T} F\;(R+Y_{,}\,D\in)\;d\in\;dy$$

$$\text{Let } G\,(R,T) = \int_R^{\infty}\int_0^T F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)$$

$$G_5\,(R_1\,T) = \left(\frac{(R-DT)^2}{2D} + \frac{\sigma^2 R}{2D^2} + \frac{\sigma^4}{4D^3}\right)F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)$$

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$$+\frac{\sqrt{\sigma^2 T}}{2}\left(T-\frac{\sigma^2}{D^3}-\frac{R}{D}\right) g\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)-\frac{\sigma^4}{4D^3} F\left(\frac{R+DT}{\sqrt{\sigma^2 T}}\right)\exp\left(\frac{2DR}{\sigma^2}\right)$$

Hence $POR = \frac{1}{Q}\left(G_5(R, L+T) - G_5(R,L) - G_5(R+Q, L+T) + G_5(R+Q, L)\right)$ (10)

Putting the various costs together we have the inventory costs for constant supply.
We have

$$C = \frac{R_C}{T} + \frac{hc}{2}\left(\frac{Q}{2} + R - DL - \frac{DT}{2}\right) + \frac{hc}{QT}(G_4(R, T+L)$$
$$- G_4 (R, L) - G_4(R+Q, T+L) + G_4 (R+Q, L))$$
$$+\frac{1}{QT}(G_2\left((R, T+L) - G_2(R,L) - G_2\ (R+Q, T+L) + G_2(R_4+Q, L)\right)$$
$$+\frac{S}{QT}\left(G_5\ (R, T+L) - G_5\ (R,L) - G_5(R+Q, L+T) + G_5(R+Q, L)\right) \quad (11)$$

The probability density function of Q, U(Q) is a gamma variate where Q is the supply

$$U(Q) = esp(-uQ)\,\frac{Q^{v-1,\mu v}}{\Gamma(v)} \qquad v, \qquad Q > 0 \quad (12)$$

$$G_4(R+Q, T) = \frac{D^2T^3}{6} - \frac{\sigma^4(R+Q)}{4D3} - \frac{DT^2(R+Q)}{2} - \frac{\sigma^2(R+Q)^2}{4D^2} + \frac{\sigma^2T^2}{4} + \frac{T(R+Q)^2}{2}$$
$$-\frac{(R+Q)^3}{6D} - \frac{\sigma^6}{8D^4}\Big) F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right) + \sigma T\frac{1}{2}\left(\frac{DT^2}{6} - \frac{T(R+Q)}{3} + \frac{(R+Q)^2}{6D} + \frac{\sigma^2T}{12D} + \frac{\sigma^2(R+Q)}{4D^2}\right.$$
$$+\frac{\sigma^4}{4D^3}\,g\left[\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right] + \frac{\sigma6}{8D^4}\,esp\left(\frac{2D(R+Q)}{\sigma^2}\right) F\left[\frac{R+Q+DT}{\sqrt{\sigma^2T}}\right] \quad (13)$$

$$G_5\ (R+Q, T) = \left[\left(\frac{\sigma^4}{2D^3} + \frac{\sigma^2}{2D^2} + \frac{(R-DT)^2}{2D}\right) + Q\left(\frac{\sigma^2}{2D^2} + \frac{2(R-DT)}{2D}\right) + \frac{Q^2}{2D}\right] F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$
$$+\frac{\sqrt{\sigma^2 T}}{2}\left(\left(T - \frac{\sigma^2}{D^2} - \frac{R}{D}\right) - \frac{Q}{D}\right) g\left[\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right] + \frac{\sigma^4}{4D^3} F\left(\frac{(R+Q+DT)}{\sqrt{\sigma^2T}}\right) \quad (14)$$

$$G_2\ (R+Q, T) = \frac{2D^2b_1}{(\sigma^2b_2^3 + 2D^2b_2^2)}\,esp\left[T\frac{(\sigma^2b_2^2+2D^2b_2)}{2D^2} - \frac{b_2(R+Q)}{D}\right]$$
$$F\left(\frac{R+Q-T\left(D+\sigma^2\frac{b_2}{D}\right)}{\sqrt{\sigma^2T}}\right) - \frac{b_1}{b_2} F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)\left[\left(T - \frac{R}{D} - \frac{\sigma^2}{2D^2} + \frac{1}{b_2}\right) - \frac{Q}{D}\right]$$
$$\frac{-\sigma^4b_1b_2^2}{2D^2(\sigma^2b_2^2+2Db_2^2)}\,esp\left(\frac{2(R+Q)}{\sigma^2}\right) F\left(\frac{R+Q-DL}{\sqrt{\sigma^2T}}\right) - \frac{2\sqrt{\sigma^2T}}{Db_2} b_1 g\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right) \quad (15)$$

$$POR = \frac{DT}{Q}\left(1 - F\left(\frac{Q-DT}{\sqrt{\sigma^2T}}\right)\right) + F\left(\frac{Q-DT}{\sqrt{\sigma^2T}}\right) - \frac{\sqrt{\sigma^2T}}{Q} g\left(\frac{Q-DT}{\sqrt{\sigma^2T}}\right)$$

Simplifying $G_4\,(R, +Q, T)$

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$$G_4(R+Q,T)=\left(\left(\frac{D^2T^3}{6}-\frac{\sigma^4R}{4D3}-\frac{DT^2R}{2}-\frac{\sigma^2R^2}{4D^2}+\frac{\sigma^2T^2}{4}+\frac{TR^2}{2}-\frac{R^3}{60}-\frac{\sigma^6}{8D^4}\right)\right)$$

$$+Q\left(-\frac{\sigma^4}{4D^3}-\frac{DT^2}{2}-\frac{2R\sigma^2}{4D^2}+\frac{2TR}{2}-\frac{3R^2}{6D}\right)+Q^2\left(\frac{\sigma^2}{4D^2}+\frac{T}{2}+\frac{3R}{6D}\right)+\left.\frac{Q^3}{6D}\right)\right)F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$

$$+QT\frac{1}{2}\left(\frac{DT^2}{6}-\frac{TR}{3}+\frac{R^2}{6D}+\frac{\sigma^2T}{12D}+\frac{\sigma^2R}{4D^2}+\frac{04}{4D^3}\right)+Q\left(-\frac{T}{3}+\frac{2R}{6D}+\frac{\sigma^2}{4D^2}\right)+\frac{Q^2}{6D}g\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$

$$+\frac{\sigma^6}{8D^4}\operatorname{esp}\left(\frac{2D(R+Q)}{\sigma^2}\right)F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$

Multiplying by $\frac{u(Q)}{Q}$ we have

$$\frac{U(Q)}{Q}G_4(R+Q,T)=\mu^v\frac{\operatorname{esp}(-UQ)}{\Gamma(v)}\left(\left(\left(\left(\frac{D^2T^3}{6}-\frac{\sigma^4R}{4D^3}-\frac{DT^2R}{2}-\frac{\sigma^2R^2}{4D^2}+\frac{\sigma^2T^2}{4}+\frac{TR^2}{2}-\frac{R^3}{6D}-\frac{\sigma^6}{8D^4}\right)Q^{v-2}\right.\right.\right.$$

$$+\frac{Q^{v-1}}{Q}\left(-\frac{\sigma^4}{4D^2}-\frac{DT^2}{2}-\frac{DT^2}{2}+\frac{R\sigma^2}{2D^2}+TR\frac{R^2}{2D}\right)+Q^V\left(-\frac{\sigma^2}{4D^2}+\frac{T}{2}-\frac{R}{2D}\right)+\left.\frac{Q^{v-1}}{6D}\right)F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$

$$+\sqrt{\sigma^2T}\left(\left(\frac{DT^2}{6}-\frac{TR}{3}-\frac{R^2}{6D}-\frac{\sigma^2T}{12D}+\frac{\sigma^2R}{4D^2}+\frac{\sigma^4}{4D^3}\right)Q^{v-2}+Q^{v-1}\left(\frac{-T}{3}+\frac{2R}{6D}-\frac{\sigma^2}{4D^2}\right)+\frac{Q^{v+1}}{6D}\right.$$

$$g\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)+\frac{\sigma^6}{8D^4}\frac{Q^{v-2}}{\Gamma(v)}\operatorname{esp}\left(-\mu Q+\frac{2D(R+Q)}{\sigma^2}\right)F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right) \quad (16)$$

Nothing that

$$\mu^v\int_0^\infty \operatorname{esp}(-\mu Q)Q^{v-1}\operatorname{esp}-\frac{1}{2}\left(\frac{R+Q-DL}{\sigma^2L}\right)^2 dQ$$

$$=\frac{\sqrt{\sigma^2L}\mu^v}{\Gamma(v)}\operatorname{esp}\left(R\mu+\mu^2\frac{\sigma^2L}{2}-DL\right)\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-2}{2}(DL-R-\mu\sigma^2L)^{v-2-2i}\left(\frac{\sigma^2L}{2}\right)^i \quad (17)$$

Nothing that

$$\int_0^\infty \exp\frac{(-\mu Q)Q^{v+\mu^v}}{\Gamma(v)}F\left(\frac{R+Q-DL}{\sqrt{\sigma^2L}}\right)dQ$$

$$=\frac{\mu^v}{\Gamma(v)}\sum_{z=i}^{v}\frac{(v-1)!}{\mu^2(v-z)!}\sum_{i=0}^{v-z/2}\binom{v-z-i}{i}(DL-R-\mu\sigma^2L)^{v-2-2i}\left(\frac{\sigma^2L}{2}\right)^i\exp\left(R\mu+\frac{\sigma^2L}{2}-D\mu L\right) \quad (18)$$

Nothing that

$$\frac{\mu^v}{\Gamma(v)}\int_0^Q Q^{v-1}\exp(-\mu Q)\exp\left(\frac{2D(R+Q)}{\sigma^2}\right)F\left(\frac{R+Q+DL}{\sqrt{\sigma^2L}}\right)dQ$$

$$=\frac{\mu^v}{\Gamma(v)}\exp\left(R\mu+\mu^2\frac{\sigma^2l}{2}-D\mu L\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-i)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^2}(v-z)!\binom{(v-z-i)}{2}(DL-R-\mu\sigma^2L)^{v-2-2i}\left(\frac{\sigma^2L}{2}\right)^i \quad (19)$$

Hence $\int_0^\infty\frac{u(Q)}{Q}G_3(R+Q,T)DQ$ applying 16,17,18

$$G_6(R,T)=\frac{\mu^v}{\Gamma(v)}\operatorname{esp}\left(R\mu+\frac{\mu^2\sigma^2T}{2}\right)+\left(\frac{D^2T^3}{6}-\frac{\sigma^2R}{4D^2}-\frac{DT^2R}{2}-\frac{\sigma^2R^2}{4D^2}+\frac{\sigma^2T^2}{4}+\frac{TR^2}{2}-\frac{R^3}{6D}-\frac{\sigma^6}{8D^2}\right)$$

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$$\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-z-1}{2}}\frac{(v-2)!}{\mu^{v}(v-1-z)!}\binom{v-z-i-1}{2}\binom{v-z-i-1}{2}(DT-R-\mu\sigma^{2}2)^{v-z-i}\left(\frac{\sigma^{2}T}{2}\right)^{i}$$
$$+\left(\frac{-\sigma^{4}}{4D^{2}}-\frac{DT^{2}}{2}-\frac{R\sigma^{2}}{2D^{2}}+TR+\frac{R^{2}}{2D}\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)!}{\mu^{2}(v-z)!}\binom{v-2-i}{i}(DT-R-\mu\sigma^{2}T)^{v-z-i}$$
$$\left(\frac{\sigma^{2}T}{2}\right)^{i}\left(\frac{\sigma^{4}}{4D^{2}}+\frac{T}{2D}-\frac{R}{2D}+\frac{R^{2}}{2D}\right)\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{(v+1-z)}{2}}\frac{v!}{\mu^{v}(v-z+1)!}\binom{v+1-Z-1}{i}$$
$$(DT-R-\mu\sigma^{2}T)^{v+1-z-2i}+\frac{1}{6D}\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{(v+1-z)}{2}}\frac{(v+1)!}{\mu^{z}(v-z+2)!}\binom{v+1-Z-i}{i}$$
$$(DT-R-\mu\sigma^{2}T)^{v+1-z-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i}\Bigg]-\frac{\sigma^{2}T\mu^{v}esp\left(R\mu+\frac{\mu^{2}\sigma^{2}T}{2}-D\mu T\right)}{\Gamma(v)}\Bigg[\Big(\frac{D^{2}T^{2}}{6}-\frac{TR}{3}+\frac{R^{2}}{6D}$$
$$+\frac{\sigma^{2}T}{12D}+\frac{\sigma^{2}R}{4D^{2}}+\frac{\sigma^{4}}{4D^{3}}\Big)\sum_{i=0}^{\frac{v-z}{2}}\binom{v-z-i}{2}(DT-R-\mu\sigma^{2}T)^{v-z-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i}+\left(-\frac{T}{3}+\frac{R}{3D}+\frac{\sigma^{2}}{4D^{2}}\right)$$

$$\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}\binom{v-1-i}{i}(DT-R-\mu\sigma^{2}T)^{v-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i}\Bigg]$$
$$(DT-R-\mu\sigma^{2}T)^{v-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i}\Bigg]+\frac{\sigma^{6}}{8D^{4}}\frac{\mu^{v}}{\Gamma(v)}esp\left(R\mu+\frac{\mu^{2}\sigma^{2}T}{2}-D\mu T\right)$$
$$\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-z)!}{\left(\mu-\frac{D}{\sigma^{2}}\right)^{2}(v-1-z)!}\binom{v-2-i}{i}(DT-R-\mu\sigma^{2}T)^{v-z-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i} \tag{20}$$

Multiplying $G_4(R+Q,T)$ by $\frac{u(Q)}{Q}$ we have $u(Q)\ G_4(R+Q,T)/Q$

From equation (4)

$$\frac{u(Q)G_{5}(R+Q,T)}{Q}=\frac{\mu^{v}esp(-\mu Q)}{\Gamma(v)}\Bigg[\left(\frac{\sigma^{4}}{2D^{3}}+\frac{\sigma^{2}}{2D^{2}}+\frac{(R-DT)^{2}}{2D}\right)Q^{v-2}+Q^{v-1}$$
$$\left(\frac{\sigma^{2}}{2D^{2}}+\frac{2(R-DT)}{2D}\right)+\frac{Q^{v}}{2D}\Bigg]F\left(\frac{R+Q-DT}{\sqrt{\sigma^{2}T}}\right)+\frac{\sqrt{\sigma^{2}T}}{2}\left(T-\frac{\sigma^{2}}{D^{2}}-\frac{R}{D}\right)Q^{v-2}-\frac{Q^{v-1}}{D}esp\frac{(-\mu Q)\mu^{v}}{\Gamma(v)}$$
$$g\left(\frac{R+Q-DT}{\sqrt{\sigma^{2}T}}\right)-\frac{Q^{4}}{4D^{3}}\frac{\mu^{v}esp(-\mu Q)}{\Gamma(v)}F\left(\frac{R+Q-DT}{\sqrt{\sigma^{2}T}}\right)Q^{v-2} \tag{21}$$

Hence we obtain

$\int_{v}^{\infty}\frac{u(Q)G_{5}(R+Q,T)dQ}{Q}$ applying 16, 17, 18

$$G_7(R,T)=\frac{\mu^{v}esp(R\mu+\frac{\mu^{2}\sigma^{2}T}{2}-D\mu T}{\Gamma(v)}\left(\frac{\sigma^{4}}{2D^{3}}+\frac{\sigma^{2}}{2D^{2}}+\frac{(R-DT)^{2}}{2D}\right)$$
$$\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-z-1}{2}}\binom{v-1-z-i}{i}\frac{v-2}{\mu^{z}(v-1-z)!}(DT-R-\mu\sigma^{2}T)^{v-1-z-2i}\left(\frac{\sigma^{2}T}{2}\right)^{2i}$$
$$+\left(\frac{\sigma^{2}}{2D^{2}}+\frac{(R-DT)}{D}\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\mu^{z}(v-z)!}\binom{v-z-i}{i}(DT-R-\mu\sigma^{2}T)^{v-z-2i}\left(\frac{\sigma^{2}T}{2}\right)^{i}$$

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$$+\frac{1}{2D}\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v+1-z}{2}}\binom{v+1-z-1}{i}\frac{v!}{\mu^{v}(v+1-2)!}(DT-R-\mu\sigma^2T)^{v+1-z-2i}\left(\frac{\sigma^2T}{2}\right)^{i}$$

$$+\frac{\sigma^2T}{2}\frac{\mu^{v}esp\left(R\mu+\frac{\mu^2\sigma^2T}{2}-D\mu T\right)}{\Gamma(v)}\left[\left(T-\frac{\sigma^2}{D^2}+\frac{R}{D}\right)\sum_{i=0}^{\frac{v-z}{2}}\binom{v-z-i}{i}(DT-R-\mu\sigma^2T)^{v-z-2}\right.$$

$$\left(\frac{\sigma^2T}{2}\right)^{i}-\frac{1}{D}\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}(DT-R-\mu\sigma^2T)^{v-1-2i}\left(\frac{\sigma^2T}{2}\right)^{i}-\frac{\sigma^4}{4D^3}\frac{\mu^{v}}{\Gamma(v)}$$

$$esp\left(R\mu+\frac{\mu^2\sigma^2T}{2}-D\mu T\right)\sum_{z=1}^{\frac{v-1}{2}}\sum_{i=0}^{\frac{v-z-1}{2}}\frac{(v-2)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^2(v-z)!}\binom{v-z-i-1}{i}$$

$$(DT-R-\mu\sigma^2T)^{v-z-2i+1}\left(\frac{\sigma^2T}{2}\right)^{i}$$

Multiplying $u(Q)$ by POR of equation 9 (22)

We have

$$U(Q)POR=\frac{\mu^{v}esp(-\mu Q)}{\Gamma(v)}DTQ^{v-2}+(Q^{v-1}-DTQ^{v-2})esp(-uQ)F\frac{(Q-DT)}{\sqrt{\sigma^2T}}$$

$$-\frac{\mu^{v}esp(\mu Q)\sqrt{\sigma^2T}\,Q^{v-2}}{\Gamma(v)} \quad (23)$$

Let PORT = $\int_{v}^{\infty}U(Q)POR\,d\,Q$. applying 16, 17

$$PORT\ \frac{DT\mu^2}{(v-1)(v-2)}+\ \mu^{v}esp\left(R\mu^{v}+\frac{\mu^2\sigma^2T}{2}-D\mu T\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)!}{\mu^{z}(v-z)!}\binom{v-z-i}{i}$$

$$(DT-R-\mu\sigma^2T)^{v--z-2i}\left(\frac{\sigma^2T}{2}\right)^{i}-DT\ \sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-z-1}{2}}\frac{(v-2)!}{\mu^{z}(v-1+z)!}\binom{v-i-z-1}{i}$$

$$(DT-R-\mu\sigma^2T)^{v-1-z-2i}\left(\frac{\sigma^2T}{2}\right)^{i}-\frac{\mu^2}{\Gamma(v)}\sigma^2T\ esp\left(R\mu^{v}+\frac{\mu^2\sigma^2T}{2}-D\mu T\right)$$

$$\sum_{i=0}^{\frac{v-z}{2}}\binom{v-z-i}{i}(DT-R-\mu\sigma^2T)^{v-z-2i}\left(\frac{\sigma^2T}{2}\right)^{i} \quad (24)$$

Multiplying $\frac{u(Q)}{Q}$ *we have*

$$\frac{G_2}{Q}(R+Q,T)u(Q)=\frac{2D^2b_1\mu^{v}Q^{v-2}}{(\sigma^2b_3^2+2D^2b_2^2)\,\Gamma(v)}esp\left[T\left(\frac{\sigma^2b_2+2D^2b_2}{2D^2}\right)-\frac{b_2R}{DT}\right]-Q\left(\mu+\frac{b_2}{D}\right)$$

$$F\left(\frac{R+Q-T\left(D+\frac{\sigma^2b_2}{D}\right)}{\sqrt{\sigma^2T}}\right)-\frac{\mu^{v}esp(\mu Q)b_1}{b_2\,\Gamma(v)}\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)\left[\left(T-\frac{R}{D}-\frac{\sigma^2}{2D^2}+\frac{1}{b_2}\right)Q^{V-2}-\frac{Q^{v-1}}{D}\right]$$

$$-\frac{\sigma^2b_1b_2^2u^{v}Q^{v-2}esp\left(\frac{2R}{\sigma^2}\right)}{2D^2(\sigma^2b_2^2+2D^2b_2)b_2}esp\left(-Q\left(\frac{2R}{\sigma^2}+\mu\right)\right)F\left(\frac{R+Q-DT}{\sqrt{\sigma^2T}}\right)$$

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$$-\frac{2\sqrt{\sigma^2 T\, b_1}}{Db_2}\frac{\mu^{v}esp(-\mu Q)Q^{v-2}}{\Gamma(v)}g\left(\frac{R+Q-DT}{\sqrt{\sigma^2 T}}\right)$$

Hence $\int_U^\infty \frac{u(Q)}{Q}G_{18}(R+Q,T)dQ$ applying 16, 17, 18

$$G_7(R,T) = 2D^2 b_2 \mu^v esp(T\left(\frac{\sigma^2 b_2 + 2D^2 b_2}{2D^2} - \frac{b_2 R}{D}\right)\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-1-z}{2}}\frac{(v-z)!}{\left(\mu-\frac{b_2}{D}\right)^2 (v-z)!}$$

$$\binom{v-1-z-i}{i}(DT-R-\mu\sigma^2 T)^{v-1-z-i}\left(\frac{\sigma^2 T}{2}\right)^i esp\left[\left(\mu+\frac{b_2}{D}\right)\sigma^2 T - \left(D+\frac{\sigma^2 b_2}{D}\right)T\right)\right]$$

$$esp\left(R\mu \quad + \frac{\mu^2\sigma^2 T}{2} - D\mu T\right) - \frac{\mu^v b_1}{b_2\,\Gamma(v)}\left(T-\frac{R}{D}-\frac{\sigma^2}{2D^2}+\frac{1}{b_2}\right)\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-1-z}{2}}\frac{(v-z)!}{\mu^z(v-2)!}$$

$$\binom{v-1-z-2}{2}(DT-R-\mu\sigma^2 T)^{v-1-2i}\left(\frac{\sigma^2 T}{2}\right)^i - \frac{1}{D}\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)!}{\mu^z(v-2)!}\binom{v-z-i}{2}$$

$$(DT-R-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i\right] - \frac{\sigma^4 b_1 b_2^2}{2D^2(\sigma^2 b_2^3 + 2D^2 b_2^2)\,\Gamma(v)}esp\left(R\mu + \frac{\mu^2\sigma^2 T}{2} - D\mu T\right)$$

$$\sum_{z=1}^{v-1}\sum_{i=0}^{\frac{v-z-1}{2}}\frac{(v-2)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^z (v-z)!}\binom{v-z-i}{i}(DT-R-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^2$$

$$-\frac{2\sigma^2 T b_1}{Db_2}\frac{\mu^v}{\Gamma(v)}esp\left(R\mu+\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)\sum_{i=0}^{\frac{v-z}{2}}\binom{v-2-i}{i}(DT-R-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i$$

Hence the inventory cost for (nQ,R,T) when the supply is random, lead time is constant with exponential backorder cost is

$$C = \frac{R_c}{1} + \frac{SPOR_T}{T} + hc\left(\frac{X}{2\mu} + R - DL - \frac{DT}{2}\right) + \frac{hc}{T}\left(\frac{\mu}{(v-1)}\right.$$

$$\left. G_4(R,T+L) - \frac{\mu}{(v-1)}G_4(R,L) - G_6(R_1 T+L) + G_6(R,L)\right) + \frac{1}{T}\left(\frac{\mu}{(v-1)}G_2(R,T+L)\right.$$

$$\left. -\frac{\mu}{(v-1)}G_2(R,L) - G_7(R,T+L) + G_7(R,L)\right) + \frac{S}{T}\left(\frac{\mu}{(v-1)}G_5(R,T+L)\right.$$

$$\left. -\frac{\mu}{(v-1)}G_5(R_1 L) - G_7(R_1 T+L) + G_7(R_1 L)\right)$$

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